Supplementary: Generalization to All-Wheel-Drive and Rear-Wheel-Drive Vehicles

This supplementary describes how REFINE can be extended to All-Wheel-Drive (AWD) and Rear-Wheel-Drive (RWD) vehicles. AWD vehicles share the same dynamics as Front-Wheel-Drive (FWD) vehicles,

$$\dot{x}(t) = \begin{bmatrix} v_{\mathrm{x}}(t)\cos h(t) - v_{\mathrm{y}}(t)\sin h(t) \\ v_{\mathrm{x}}(t)\sin h(t) + v_{\mathrm{y}}(t)\cos h(t) \\ r(t) \\ \frac{1}{m}\big(F_{\mathrm{xf}}(t) + F_{\mathrm{xr}}(t)\big) + v_{\mathrm{y}}(t)r(t) + \Delta_{v_{\mathrm{x}}}(t) \\ \frac{1}{m}\big(F_{\mathrm{yf}}(t) + F_{\mathrm{yr}}(t)\big) - v_{\mathrm{x}}(t)r(t) + \Delta_{v_{\mathrm{y}}}(t) \\ \frac{1}{I_{\mathrm{zz}}}\big(l_{\mathrm{f}}F_{\mathrm{yf}}(t) - l_{\mathrm{r}}F_{\mathrm{yr}}(t)\big) + \Delta_{r}(t) \end{bmatrix}. \tag{1}$$

with one exception. In an AWD vehicle, only the lateral rear tire force is estimated and all the other three tire forces are controlled by wheel speed and steering angle. In particular, computations related to the lateral tire forces as

$$\begin{aligned} F_{\mathrm{yf}}(t) = &- \frac{I_{\mathrm{zz}}K_r}{l_{\mathrm{f}}}\big(r(t) - r^{\mathrm{des}}(t,p)\big) - \frac{I_{\mathrm{zz}}K_h}{l_{\mathrm{f}}}\big(h(t) - h^{\mathrm{des}}(t,p)\big) + \\ &+ \frac{I_{\mathrm{zz}}}{l_{\mathrm{f}}}\dot{r}^{\mathrm{des}}(t,p) + \frac{l_{\mathrm{r}}}{l_{\mathrm{f}}}F_{\mathrm{yr}}(t) + \frac{I_{\mathrm{zz}}}{l_{\mathrm{f}}}\tau_r(t,p), \end{aligned} \tag{2}$$

and

$$\begin{aligned} \alpha_{\mathrm{f}}(t) = &- \frac{I_{\mathrm{zz}}K_r}{l_{\mathrm{f}}c_{\alpha\mathrm{f}}}\big(r(t) - r^{\mathrm{des}}(t,p)\big) - \frac{I_{\mathrm{zz}}K_h}{l_{\mathrm{f}}c_{\alpha\mathrm{f}}}\big(h(t) - h^{\mathrm{des}}(t,p)\big) + \\ &+ \frac{I_{\mathrm{zz}}}{l_{\mathrm{f}}c_{\alpha\mathrm{f}}}\dot{r}^{\mathrm{des}}(t,p) + \frac{l_{\mathrm{r}}}{l_{\mathrm{f}}c_{\alpha\mathrm{f}}}F_{\mathrm{yr}}(t) + \frac{I_{\mathrm{zz}}}{l_{\mathrm{f}}c_{\alpha\mathrm{f}}}\tau_r(t,p). \end{aligned} \tag{3}$$

are identical to the FWD case. However, both the front and rear tires contribute nonzero longitudinal forces, and they can be specified by solving the following system of linear equations:

$$\begin{aligned} l_{\mathrm{f}}F_{\mathrm{xf}}(t) &= l_{\mathrm{r}}F_{\mathrm{xr}}(t) \\ F_{\mathrm{xf}}(t) + F_{\mathrm{xr}}(t) &= -mK_{v_{\mathrm{x}}}v_{\mathrm{x}}(t) + mK_{v_{\mathrm{x}}}v_{\mathrm{x}}^{\mathrm{des}}(t,p) + \\ &\quad + m\dot{v}_{\mathrm{x}}^{\mathrm{des}}(t,p) - mv_{\mathrm{y}}(t)r(t) + m\tau_{v_{\mathrm{x}}}(t,p) \end{aligned} \tag{4}$$

Longitudinal tire forces $F_{\mathrm{xf}}(t)$ and $F_{\mathrm{xr}}(t)$ computed from (4) can then be used to compute wheel speed $\omega_{\mathrm{f}}(t) = \omega_{\mathrm{r}}(t)$ as in

$$\omega_{\mathrm{f}}(t) = \begin{cases} \left(\dfrac{lF_{\mathrm{xf}}(t)}{\mu m g l_{\mathrm{r}}} + 1\right)\dfrac{v_{\mathrm{x}}(t)}{r_{\mathrm{w}}}, & \text{during braking} \\ \dfrac{v_{\mathrm{x}}(t)}{\left(1 - \frac{lF_{\mathrm{xf}}(t)}{\mu m g l_{\mathrm{r}}}\right)r_{\mathrm{w}}}, & \text{during acceleration.} \end{cases} \tag{5}$$

In the meantime, the computation of slip ratios needs to be modified to

$$\lambda_{\mathrm{f}}(t) = \lambda_{\mathrm{r}}(t) = \frac{1}{g\mu}\big(-K_{v_{\mathrm{x}}}v_{\mathrm{x}}(t) + K_{v_{\mathrm{x}}}v_{\mathrm{x}}^{\mathrm{des}}(t,p) + \dot{v}_{\mathrm{x}}^{\mathrm{des}}(t,p) - v_{\mathrm{y}}(t)r(t) + \tau_{v_{\mathrm{x}}}(t,p)\big) \tag{6}$$

to verify Assumption 2 of our main text along the longitudinal direction. Compared to FWD, in RWD the longitudinal front tire force is 0 and the longitudinal rear tire force is controlled. Thus one can generalize to RWD by switching all related computations on $F_{\mathrm{xf}}(t)$ and $F_{\mathrm{xr}}(t)$ from the FWD case.